Example Question 1 Answer:

This an ESS problem with the following values: Player A (Wrasse 1) Player B (Wrasse 2)

Strategy 1: groom (remove parasites) and Strategy 2: bite (remove skin)

Payoffs (expected gain in # of eggs):

P11 (both groom) = 8 eggs

P12 (Wrasse 1 grooms and Wrasse 2 bites)= Expected eggs gained from grooming (8) x probability of not being chased (0.80) + expected eggs if Wrasse 1 is chased 0 x prob of being chased (.2) = 8(0.80) + 0 = 6.4

P21 (Wrasse 1 bites and Wrasse 2 grooms): Expected eggs from biting (15) x prob of not being chased (.90)) + expected eggs if Wrasse 1 is chased (0)) = 15(.9) + 0 = 13.5

P22 (both bite) = Expected eggs from biting (15) times prob. of not getting chased (0.6) + eggs if chased (0) = 15(.6) + 0 = 9

	Groom	*Bite*
Groom	*8*	*6.4*
Bite	*13.5*	*9*

Since P21 > P11 and P22 > P12 you get a classic type II stable ESS for Strategy 2 (Bite) when you plug these inequalities into the 2x2 payoff matrix (see the circles). Effectively, the increased expectation of egg output from biting compensates for the increased risks of being chased that are associated with that tactic.

Part 2: When the risk of being chased for both wrasses biting increases to 80%, the payoff, P22 decreases to 15(0.2) = 3. Now, P22 < P12, generating a prediction for a mixed, stable ESS. The frequency for "groom" can be calculated from the equation:

(P12 - P22)/((P12 - P22)+(P21 - P11))

This yields the following:

8	*6.4*
13.5	*3*

(6.4-3)/[(6.4-3)+(13.5-8)] = 3.4/8.9 = 0.38.2 or:
"groom" 38.2 % of the time and "bite" 61.8 % of the time.

Example Question 2 Answer:

This is another classic 2x2 symmetrical game theory problem:

	Forage	Chase
Forage	F	$F-B$
Chase	$F+B-C_1$	$F+C_1-C_2$

Let ***F = Expected gain of foraging*** ***B = the gain from chasing***
C_1 = loss from chasing a forager ***C_2 = loss from chasing a chaser***

Where $F = 400$, $B = 300$, $C_1 = 120$, $C_2 = 200$

Payoffs would be calculated as follows:

$P_{11} = F = 400$
$P_{12} = F - B = 400 - 300 = 100$
$P_{21} = F + B - C = 400 + 300 - 120 = 580$
$P_{22} = F - C1 - C2 = 400 - 120 - 200 = 80$

400	100
580	80

Plugging these values into the Payoff matrix reveals a mixed, stable ESS

The frequency of playing a "forage" strategy at equilibrium can be calculated as:

$f = (P_{12}-P_{22})/((P_{12}-P_{22})+(P_{21}-P_{11}))$
$= (100-80)/((100-80)+(580-400))=20/200=0.1$ or $1/10$.

The question asked about the predicted number of birds chasing. This is given my multiplying the frequency of chasing (1-f) by the size of the population (1000). This value = (1 - 0.1) * 1000 = 0.9 * 1000 = 900 birds.

If fish densities increased forager success tenfold, to 4000, then the Payoff matrix would be:

4000	1000
6880	3680

This is based on the following calculations:

$P_{11} = F = 4000$
$P_{12} = F - B = 4000 - 3000 = 1000$
$P_{21} = F + B - C = 4000 + 3000 - 120 = 6{,}880$
$P_{22} = F - C1 - C2 = 4000 - 120 - 200 = 3{,}680$

Now a Pure ESS for "Chase" is predicted… all birds should chase as a strategy to gain food.

Example Question 3 Answer:

This is yet another classic 2x2 symmetrical game theory problem with the following values:

	Fight	***Display***
Fight	P_{11}	P_{12}
Display	P_{21}	P_{22}

-50	***800***
0	***400***

	●
●	

Payoffs (expected total number of eggs produced):

P_{11} (both fight) = (prob of winning) x (payoff of winning - cost of fighting) + (prob of losing) x (payoff of losing - cost of fighting)

Where: payoff to winner = 800; prob of winning = 0.5; payoff to loser = 0; prob of losing =0.5

Cost of fighting = 0.75 x - 600 = - 450 (reduction in egg production)

So P_{11} = 0.5(800 - 450) + 0.5(0 – 450) = 175 – 225 = -50

P_{12} (one fights, opponent displays) = prob of winning x ben of winning = 1 x 800 = 800

P_{21} (one displays, opponent fights) = prob of winning x ben of winning = 0 x 800 = 0

P_{22} (both display) = (prob of sharing x ben of sharing) + prob of getting all x ben of getting all) + prob of leaving x ben of leaving) = .5(400) + .25(800) + .25(0) = 200 + 200 = 400

Plugging these payoffs into the matrix we find a mixed stable strategy for fighting and displaying. The frequency of fighting can be calculated from the equation: $f = (P_{12}-P_{22})/((P_{12}-P_{22})+(P_{21}-P_{11}))$

For the payoffs given, f = (800-400)/((800-400)+(0-(-50))=400/450=0.889 or 8/9.

If the risk of losing a claw drops to 20% then the cost of fighting is now .2(-600) = -120

Thus, P_{11} becomes .5(800 - 120) + .5(0 – 120) = 340 – 60 = 280 and a pure strategy for "fight" is predicted.

280	***800***
0	***400***

●	●

Example Question 4 Answer:

Finally, another optimality problem!

To answer this question, you must first calculate the expected clutch success, irrespective of predation on the clutch. This would be the sum of the products of each clutch size multiplied by its probability of occurrence.

For Habitat I: 4(0.6) + 3(0.3) + 2(0.1) + 1(0) = 2.4 + 0.9 + 0.2 = 3.5
For Habitat II: 4(0.1) + 3(0.3) + 2(0.5) + 1(0.1) = 0.4 + 0.9 + 1.0 + 0.1 = 2.4

Next these values must be multiplied by the probability of not being eaten by predator. If p is the probability of predation then (1 – p) is the probability of no predation.

For Habitat I, p = 0.4, and 1-p = 0.6, 3.5(0.6) = 2.1 This is the expected success in Habitat I.
For Habitat II, p = 0.05, and 1-p = 0.95, 2.4(0.95) = 2.28 This is the expected success in Habitat II.

Thus, Habitat II would be a better choice.